Permanent Market Freeze in a Decentralized Asset Market*

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Abstract

When a decentralized asset market is subject to adverse selection, trading of high-quality assets can cease due to self-fulfilling expectations. Following such a partial market freeze, sellers of high-quality assets accumulate in the market. Thus, the average quality of assets on sale increases. On the one hand, this renders buyers more willing to switch to offering high prices, which is a precondition for trade of all assets to resume. But on the other hand, as a buyer is more likely to acquire a high-quality asset, high-valuation holders of low-quality assets may wish to sell their assets and enter the pool of buyers. If additional holders of low-quality assets become sellers, the average quality of assets on sale falls. Consequently, buyers may no longer be willing to offer high prices. Thus, a partial market freeze can be a trap from which no transition path along which all assets are traded exists.

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1 Introduction

When trade is decentralized, expectations about future trading opportunities crucially shape the ensuing pattern of trade. Namely, there is scope for coordination failures, as shown by Diamond (1982) in a production economy. In

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addition to multiple steady states, an environment with decentralized trade can support nontrivial equilibrium dynamics, as illustrated by Diamond and Fudenberg (1989) in a further exploration of Diamond (1982).

Recent contributions by Duffie et al. (2005) and Lagos and Rocheteau (2009) introduce decentralized trading into asset market models. Although Lagos and Rocheteau (2009) along with Chiu and Koeppl (2011) and Hellwig and Zhang (2012), both extensions of Duffie et al. (2005), study transitional dynamics, possible path-dependencies in equilibrium dynamics have not been analyzed in this strand of the literature. More specifically, the question of whether self-fulfilling expectations can support a transition both into and out of a market freeze has not been addressed. To answer this question, I investigate how, in a model of a decentralized asset market with asymmetric information, the existence of different equilibrium paths depends on past patterns of trade in the market.

I study an environment where agents have asymmetric information about the type of a durable asset. Trade of the asset occurs in a decentralized market, where buyers and sellers are randomly matched. In a matched buyer-seller pair, the buyer makes a take-it-or-leave-it offer for the seller's asset without knowing its type. The ease of finding a counterparty depends on the ratio of buyers to sellers in the market. In this environment, a buyer's willingness to pay for an asset is increasing in the price they expect to be able to sell the asset at in the future. Thus, buyers' price offers display strategic complementarity, giving rise to multiple equilibria. Namely, there exists a high-price equilibrium where all types of the asset are traded and a low-price equilibrium where only the low-quality assets are traded, i.e. a partial market freeze. Moreover, the market can freeze due to self-fulfilling expectations. When only the low-quality assets are traded, sellers of high-quality assets accumulate in the market. Thus, if trade of all assets resumed, a buyer would encounter sellers of high-quality assets more frequently. Consequently, more owners of low-quality assets may wish to sell their asset and enter the pool of buyers. But an increase in the fraction of sellers with low-quality assets decreases buyers' willingness to switch to offering high prices, which would resume the trade of all assets. Therefore, recovery from a market freeze can be precluded by the fall in the average quality of assets which would occur if trade of all assets was to resume.

This paper is most closely related to the literature on market freezes in asset markets with search frictions.[1] Chiu and Koeppl (2011) study optimal intervention in a market frozen due to a shock to asset quality. As in the present analysis,

[1] Rocheteau and Weill (2011) provide a survey of the literature on asset markets with search frictions.

the environment in Chiu and Koeppl (2011) supports multiple steady states due to buyers' price offers exhibiting strategic complementarity. However, Chiu and Koeppl (2011) do not analyze market dynamics induced by a self-fulfilling market freeze. Moreover, preference shocks in Chiu and Koeppl (2011) are not independent of an agent's asset holdings. Consequently, the scope for self-sustaining recoveries from a market freeze is limited. Hellwig and Zhang (2012) investigate self-fulfilling market freezes and find that a freeze can be preceded by a fire sale. But unlike the present study, Hellwig and Zhang (2012) contains no analysis of self-fulfilling recoveries from a freeze. Lagos et al. (2011) establish that search frictions can discourage dealers from providing liquidity during crises, justifying government intervention. While Lagos et al. (2011) builds on Lagos and Rocheteau (2009), both Chiu and Koeppl (2011) and Hellwig and Zhang (2012) augment the environment in Duffie et al. (2005) with asymmetric information. Similarly, the environment in this paper descends from that in Duffie et al. (2005).

More generally, the present study contributes to the literature concerned with the implications of asymmetric information for decentralized trade. Within this line of work, Moreno and Wooders (2002) and Moreno and Wooders (2010) are the closest to this paper.[2] Moreno and Wooders (2002) study trading patterns and market compositions in a one-time entry model. Moreno and Wooders (2010), in turn, analyze an environment in which new agents enter the market in each period. Both of these papers are concerned with nondurable goods while the present study investigates trade of durable goods within a constant population of agents.

In an asset market context, Camargo and Lester (2011) study how a market subject to adverse selection clears over time in a one-time entry model with random matching. Guerrieri and Shimer (2012), on the other hand, analyze asset prices and trading probabilities in a competitive search framework. Similarly, Chang (2012) shows that traders' valuations being private information can lead to fire sales in an environment with competitive search. These three papers study environments where, unlike in the present study, there is no scope for coordination failures. This paper contributes to the literature on decentralized trade in the presence of asymmetric information by showing how a market can freeze but not recover due to self-fulfilling expectations.

[2]Other more distantly related papers include Wolinsky (1990), Serrano and Yosha (1993), Williamson and Wright (1994), Blouin and Serrano (2001) and Blouin (2003) to name but a few.

2 Environment

Time is continuous and the economy infinitely lived. There are two consumption goods, perishable fruit and a numéraire good. There is a measure A of durable assets, which are of two types, peaches[3] and lemons. Each peach yields δ_g units of fruit per period while each lemon yields δ_b units of fruit per period, with $\delta_g > \delta_b$. Proportion λ of the assets are peaches.

Agents are infinitely lived, risk-neutral and discount future payoffs with a discount rate of r. Measure A of agents are initially endowed with 1 unit of the asset each while a measure 1 of agents initially hold no assets. Agents can hold either 0 or 1 units of the asset at any point in time. This restriction on asset holdings keeps the distribution of assets among agents tractable. The type of an asset is private information to its current owner. Each agent is in one of two states at any point in time, having either low or high valuation for the asset. The instantaneous utility of an agent with high valuation from a type-i asset is δ_i, whilst that of an agent having a low valuation is $\delta_i - x$, where $i \in \{b, g\}$. The parameter x satisfies $x < \delta_b$, ensuring that agents never want to dispose of their asset. An agent with high valuation transits to the state of low valuation with intensity κ and transits back with intensity ν.[4] An agent's valuation state is their private information. Each agent can produce any amount of the numéraire good. An agent's instantaneous utility from a net consumption $c \in \mathbb{R}$ of the numéraire good is additive to that from fruit and equal to c.

Trade is decentralized and takes place between an agent without an asset but wishing to buy one and an agent wishing to sell their asset.[5] Buyers and sellers are bilaterally and randomly matched such that the meeting rate between buyers and sellers is $\mu\gamma_B\gamma_S/(\gamma_B + \gamma_S)$, where γ_B and γ_S denote the measures of buyers and sellers, respectively.[6] That is, a buyer meets sellers at intensity $\mu\gamma_S/(\gamma_B + \gamma_S)$ and a seller meets buyers at intensity $\mu\gamma_B/(\gamma_B + \gamma_S)$. The parameter μ determines the degree of search friction in the market. When $\mu \to \infty$, trade becomes frictionless. In a matched buyer-seller pair, the buyer makes a take-it-or-leave-it offer for the seller's asset. This ensures that the price offer does not contain information about the type of the seller's asset.[7] To characterize patterns of trade, it is useful to label the agents according to their status, namely

[3] In what follows, I will use the terms good assets and peaches interchangeably.

[4] That is, a high-valuation (low-valuation) agent receives a valuation shock with Poisson arrival rate κ (rate ν).

[5] As two agents swapping assets cannot make both of them better off, it is ruled out by assumption. Thus, given the restriction on asset holdings, only agents without assets can be buyers.

[6] Stevens (2007) provides microfoundations for this matching function.

[7] If the seller was to suggest a price or could make a counteroffer, they could potentially signal the type of their asset with the price.

whether they are in the state of low (l) or high (h) valuation and whether they hold a lemon (b), a peach (g) or no asset (n). Thus, the set of agent statuses is $\{lb, hb, lg, hg, ln, hn\} =: \mathscr{J}$.

3 Equilibrium

Scope for trade in this environment arises from the switches in each agent's valuation state, inducing different agents to value both types of the asset differently at any point in time. In the absence of private information, trade would occur between high-valuation agents without assets and low-valuation agents with assets. When both an owner's valuation state and the type of their asset are private information, also owners of lemons who are in the state of high valuation may wish to sell their asset if buyers offer prices accepted by sellers of good assets.[8] However, among the agents without assets only those with high valuation are willing to buy an asset. That is, $\gamma_B = \gamma_{hn}$ and either $\gamma_S = \gamma_{lb} + \gamma_{lg}$ or $\gamma_S = \gamma_{lb} + \gamma_{lg} + \gamma_{hb}$. Figure 1 illustrates the flows between pools of agents with different statuses due to valuation shocks and trade. Note that the measures of agents with different asset holdings are constant over time as no assets are disposed of and agents hold either 0 or 1 units of the asset at any point in time. The measures of agents in states of high and low valuation, on the other hand, depend both on the intensities of the valuation shocks and on trade. When all assets are traded and buyers are matched with sellers at a high rate, the measures of low-valuation owners and that of high-valuation nonowners are low. Similarly, when agents transit faster to the state of high valuation than to that of low valuation, i.e. $\nu > \kappa$, the measures of low-valuation agents are depressed.

In the remainder of this section I first discuss agents' problems. Then, I specify evolution of measures of agents with different statuses and define equilibrium. Finally, I delve into equilibrium patterns of trade and characterize stationary equilibria.

3.1 Agents' problems

In what follows, owners of assets who are not willing to sell their asset will be referred to as holders. That is, at any point in time an owner is either a seller or a holder. Let V_j denote the value function of an agent with status $j \in \mathscr{J}$. A high-valuation holder of a type-i asset derives instantaneous utility δ_i and transits to the state of low valuation at a random time τ_d, where $\tau_d - t$ is exponentially

[8] If valuation states were public information, a high-valuation seller of a lemon could not conceal their asset's type from buyers as no high-valuation owner of a good asset wants to sell their asset.

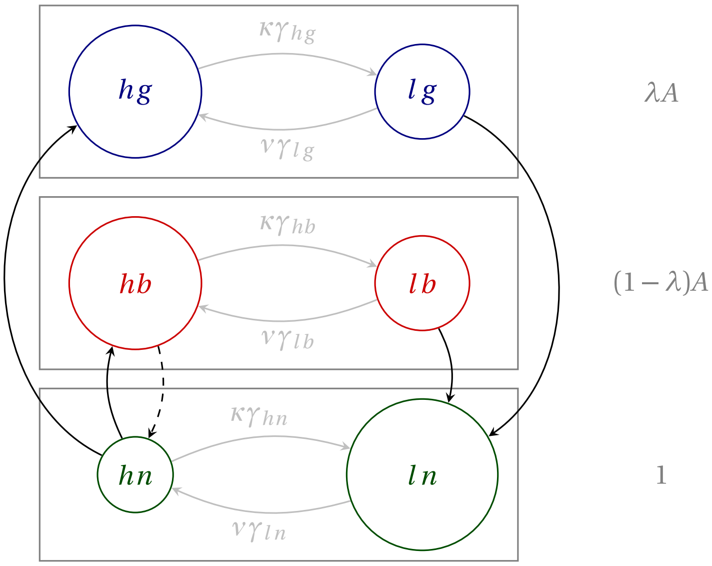

Figure 1: Flows between pools of agents with different statuses due to valuations shocks (gray) and trade (black). The dashed line represents an additional flow due to trade when also high-valuation owners of lemons are willing to sell their asset. The labels on the right indicate the measures of agents with different asset holdings (represented by the three rectangles).

distributed with mean $1/\kappa$. Thus, the value function of a high-valuation holder of a type-i asset V_{hi} satisfies

$$V_{hi}(t) = \mathbb{E}_t\left[\int_t^{\tau_d} e^{-r(s-t)}\delta_i\,\mathrm{d}s + e^{-r(\tau_d-t)}V_{li}(\tau_d)\right]. \tag{1}$$

A low-valuation seller of a type-i asset obtains utility $\delta_i - x$ until they either meet a buyer and sell their asset or transit to the state of high valuation. Denoting the next time at which the seller meets a buyer with τ_B and the next time at which the low-valuation agent's valuation state changes with τ_u, the seller's value function V_{li} becomes

$$\begin{aligned} V_{li}(t) = \mathbb{E}_t\Bigg[&\int_t^{\tau} e^{-r(s-t)}(\delta_i - x)\,\mathrm{d}s + e^{-r(\tau_u-t)}V_{hi}(\tau_u)\mathbb{1}_{\{\tau_u=\tau\}} \\ &+ e^{-r(\tau_B-t)}W_{li}(\tau_B)\mathbb{1}_{\{\tau_B=\tau\}}\Bigg], \end{aligned} \tag{2}$$

where $\tau = \min\{\tau_B, \tau_u\}$. Denoting the distribution function of prices offered by buyers at time t with F_t, the value function of a low-valuation seller of a type-i

asset matched with a buyer W_{li} satisfies

$$W_{li}(\tau) = \int \max\{p + V_{ln}(\tau), V_{li}(\tau)\} \,\mathrm{d}F_\tau(p). \tag{3}$$

Analogously, the value function a high-valuation seller of a type-i asset is given by[9]

$$\begin{aligned} V^S_{hi}(t) = \mathbb{E}_t \Bigg[\int_t^\tau & e^{-r(s-t)} \delta_i \,\mathrm{d}s + e^{-r(\tau_d - t)} V_{li}(\tau_d) \mathbb{1}_{\{\tau_d = \tau\}} \\ & + e^{-r(\tau_B - t)} W_{hi}(\tau_B) \mathbb{1}_{\{\tau_B = \tau\}} \Bigg], \end{aligned} \tag{4}$$

where $\tau = \min\{\tau_B, \tau_d\}$ and

$$W_{hi}(\tau) = \int \max\{p + V_{hn}(\tau), V_{hi}(\tau)\} \,\mathrm{d}F_\tau(p). \tag{5}$$

A high-valuation buyer, currently without an asset, derives instantaneous utility 0 and experiences a change in their future expected utility when they either meet a seller and buy an asset or transit to the state of low valuation. Hence, denoting the next time at which the buyer meets a seller with τ_S and the next time at which the high-valuation agent's valuation decreases with τ_d, the buyer's value function satisfies

$$V_{hn}(t) = \mathbb{E}_t \left[e^{-r(\tau_d - t)} V_{ln}(\tau_d) \mathbb{1}_{\{\tau_d < \tau_S\}} + e^{-r(\tau_S - t)} W_{hn}(\tau_S) \mathbb{1}_{\{\tau_S < \tau_d\}} \right]. \tag{6}$$

A buyer who is matched with a seller decides on a price to offer depending on the probabilities of obtaining an asset of either quality conditional on the offered price. Denoting the probability of acquiring a type-i asset at time τ with $\sigma_{i,\tau}$, the value function of a high-valuation buyer matched with a seller becomes

$$\begin{aligned} W_{hn}(\tau) = \max_{p_\tau} \{ & \sigma_{h,\tau}(p_\tau) \left[V_{hg}(\tau) - p_\tau \right] + \sigma_{l,\tau}(p_\tau) \left[V_{hb}(\tau) - p_\tau \right] \\ & + \left[1 - \sigma_{h,\tau}(p_\tau) - \sigma_{l,\tau}(p_\tau) \right] V_{hn}(\tau) \}. \end{aligned} \tag{7}$$

Finally, an agent holding no asset and being in the state of low valuation obtains instantaneous utility 0 and experiences an increase in their valuation at a random time τ_u.[10] Thus,

$$V_{ln}(t) = \mathbb{E}_t \left[e^{-r(\tau_u - t)} V_{hn}(\tau_u) \right]. \tag{8}$$

[9] In equilibrium, high-valuation owners of good assets are always holders. Thus, (1) always applies for $i = g$.

[10] In equilibrium, a low-valuation agent without an asset never wishes to buy an asset.

3.2 Evolution of measures of agents

The exogenous transitions between the two valuation states and agents' trading strategies determine the flows between the pools of agents with different statuses. At time t, high-valuation owners transit to the state of low valuation at rate κ and low-valuation owners transit to the state of high-valuation at rate ν. Moreover, as a result of each successful trading encounter a high-valuation buyer becomes a high-valuation owner. Thus, the laws of motion of high-valuation agents satisfy[11]

$$\dot{\gamma}_{hi}(t)=\nu\gamma_{li}(t)-\kappa\gamma_{hi}(t)+\frac{\mu\gamma_{hn}(t)\gamma_{li}(t)}{\gamma_{hn}(t)+\gamma_S(t)}\int_{\mathscr{P}_i}\mathrm{d}F_\tau(p) \tag{9}$$

$$\dot{\gamma}_{hn}(t)=\nu\gamma_{ln}(t)-\kappa\gamma_{hn}(t)-\frac{\mu\gamma_{hn}(t)}{\gamma_{hn}(t)+\gamma_S(t)}\sum_{i=b,g}\gamma_{li}(t)\int_{\mathscr{P}_i}\mathrm{d}F_\tau(p), \tag{10}$$

where $i\in\{b,g\}$ and $\mathscr{P}_i$ denotes the set of prices accepted by low-valuation sellers of type-i assets. The integrals represent the fraction of encounters between buyers and low-valuation sellers of type-i assets resulting in trade. As by construction $\gamma_{ln}+\gamma_{hn}=1$, $\gamma_{lg}+\gamma_{hg}=\lambda A$ and $\gamma_{lb}+\gamma_{hb}=(1-\lambda)A$, implying that $\dot{\gamma}_{li}(t)=-\dot{\gamma}_{hi}(t)$ and $\dot{\gamma}_{ln}(t)=-\dot{\gamma}_{hn}(t)$, it is sufficient to know the measures of high-valuation agents to evaluate the evolution of measures of agents.

3.3 Definition of equilibrium

Having specified the problems of agents with different statuses and the evolution of measures of agents, let me define equilibrium in this environment. As the value function of a high-valuation seller (4) collapses to that of a high-valuation holder (1) when high-valuation owners choose not to trade, the following definition does not require agents' strategies to satisfy (1).

Definition 1. *Given initial measures of agents, an equilibrium is a time path for trading strategies and measures of agents such that*

1. *Each agent's trading strategy satisfies* (2), (3), (4), (5), (6), (7) *and* (8).

2. *Evolution of measures of agents satisfies* (9) *and* (10).

[11]These laws of motion presuppose that all sellers employ the same strategy, which will below be shown to be true in equilibrium. Also note that a high-valuation owner selling their asset does not alter the number of high-valuation owners. But when high-valuation owners of lemons become sellers, increasing γ_S, the meeting rate between buyers and sellers falls.

3.4 Trade

Let me begin analyzing equilibrium patterns of trade by considering sellers. Inspecting (3) reveals that sellers employ reservation price strategies. Moreover, one can see that all low-valuation sellers of a type-i asset at time t have the reservation price $V_{li}(t) - V_{ln}(t) =: \bar{p}_{li}(t)$. Similarly, the reservation price of a high-valuation seller at time t is $V_{hi}(t) - V_{hn}(t) =: \bar{p}_{hi}(t)$. That is, the offered price should at least compensate for relinquishing the asset.

Consider next the problem of a buyer choosing which price to offer. If a buyer offered $\bar{p}_{lb}(t)$, only low-valuation sellers of lemons would accept. But the buyer would capture all the surplus, arising from the seller attaching a lower value to the dividend flow than the buyer. To obtain a good asset, a buyer would need to offer at least $\bar{p}_{lg}(t)$. However, given that low-valuation sellers of lemons also accept this price, the buyer would risk obtaining a lemon. Moreover, it is possible that when buyers offer $\bar{p}_{lg}(t)$, also high-valuation owners of lemons prefer selling their asset to holding it. Thus, whether the expected surplus to a buyer from offering $\bar{p}_{lg}(t)$ is higher than that from offering $\bar{p}_{lb}(t)$ depends on which agents are willing to sell at that price and on the composition of assets in the market. However, offering any other price than $\bar{p}_{lb}(t)$ or $\bar{p}_{lg}(t)$ is not optimal since such offers would only reduce the expected surplus accruing to the buyer.

To understand how buyers choose between offering the low price $\bar{p}_{lb}(t)$ and the high price $\bar{p}_{lg}(t)$, consider a buyer's expected surplus. Also, let $\tilde{\lambda} := \gamma_{lg}/(\gamma_{lg}+\gamma_{lb})$, the fraction of low-valuation owners with good assets. Given that $\bar{p}_{lg}(t)$ is accepted by all sellers, the probability of obtaining a good asset when offering $\bar{p}_{lg}(t)$ is equal to the fraction of sellers holding good assets, γ_{lg}/γ_S.[12] Then, a buyer's expected surplus from offering $\bar{p}_{lg}(t)$ at time t is

$$\Gamma(\bar{p}_{lg}(t)) = \frac{\gamma_{lg}(t)}{\gamma_S(t)} V_{hg}(t) + \left[1 - \frac{\gamma_{lg}(t)}{\gamma_S(t)}\right] V_{hb}(t) - [V_{lg}(t) - V_{ln}(t)] - V_{hn}(t). \quad (11)$$

This expression shows that a buyer offering $\bar{p}_{lg}(t) = V_{lg}(t) - V_{ln}(t)$ obtains a good asset with probability γ_{lg}/γ_S and a lemon with the complementary probability $1 - \gamma_{lg}/\gamma_S$. The buyer's outside option is to remain a high-valuation nonowner, the value of which is $V_{hn}(t)$. If the buyer was to offer $\bar{p}_{lb}(t)$ instead, only low-valuation owners of lemons would accept. Thus, the buyer risks having their offer rejected and having to wait to meet another seller. Consequently, the

[12]Note that $\gamma_{lg}/\gamma_S = \tilde{\lambda}$ when only low-valuation owners are willing to sell their asset, i.e. $\gamma_S = \gamma_{lb} + \gamma_{lg}$.

expected surplus from offering $\bar{p}_{lb}(t)$ is[13]

$$\Gamma(\bar{p}_{lb}(t)) = \frac{\gamma_{lb}(t)}{\gamma_S(t)}[V_{hb}(t) - [V_{lb}(t) - V_{ln}(t)] - V_{hn}(t)]. \tag{12}$$

To gain further insight into when both types of the asset and when only lemons will be traded, consider the value functions of agents with different statuses. Differentiating (1), (2), (6) and (8) with respect to t and rearranging, one obtains the following Hamilton-Jabobi-Bellman equations[14]

$$rV_{hi}(t) = \delta_i + \kappa[V_{li}(t) - V_{hi}(t)] + \dot{V}_{hi}(t) \tag{13}$$

$$rV_{li}(t) = \delta_i - x + \nu[V_{hi}(t) - V_{li}(t)] + m_{hn}(t)[W_{li}(t) - V_{li}(t)] + \dot{V}_{li}(t) \tag{14}$$

$$rV_{hn}(t) = \kappa[V_{ln}(t) - V_{hn}(t)] + m_S(t)[W_{hn}(t) - V_{hn}(t)] + \dot{V}_{hn}(t) \tag{15}$$

$$rV_{ln}(t) = \nu[V_{hn}(t) - V_{ln}(t)] + \dot{V}_{ln}(t), \tag{16}$$

where $m_{hn} = \mu\gamma_{hn}/(\gamma_{hn} + \gamma_S)$ and $m_S = \mu\gamma_S/(\gamma_{hn} + \gamma_S)$, the probabilities of a seller meeting a buyer and of a buyer meeting a seller, respectively. Let me first solve for the value functions of agents holding good assets. Note that in any equilibrium a low-valuation owner of a good asset receives no surplus, i.e. $W_{lg} = V_{lg}$, as either all buyers offer $\bar{p}_{lb}(t)$ and good assets are not traded or at least some buyers offer $\bar{p}_{lg}(t)$, the price at which low-valuation owners of good assets are indifferent between selling and holding. Thus, the value of owning a good asset is equal to the value of holding the asset forever. Consequently, in any equilibrium, the value functions of agents with good assets are time-invariant and given by

$$V_{hg} = \frac{\delta_g}{r} - \left(\frac{\kappa}{\kappa + \nu + r}\right)\left(\frac{x}{r}\right) \tag{17}$$

$$V_{lg} = \frac{\delta_g}{r} - \left(\frac{\kappa + r}{\kappa + \nu + r}\right)\left(\frac{x}{r}\right). \tag{18}$$

One observes that the value of owning a good asset is determined by the dividend flow and the expected utility loss from valuation shocks. Moreover, the values are independent of the distribution of prices offered by buyers. Next, consider the value functions of agents with lemons. From (13) and (14), one

[13] The fraction $\gamma_{lb}(t)/\gamma_S(t)$ is equal to $1 - \tilde{\lambda}$ when only low-valuation owners are willing to sell their asset, i.e. $\gamma_S = \gamma_{lb} + \gamma_{lg}$.

[14] Note that the value functions satisfy these equations when only low-valuation owners are willing to sell their asset. When also high-valuation owners of lemons are sellers, V_{hb} satisfies $rV_{hb}(t) = \delta_b + \kappa[V_{lb}(t) - V_{hb}(t)] + m_{hn}(t)[W_{hb}(t) - V_{hb}(t)] + \dot{V}_{hb}(t)$.

obtains

$$
\begin{aligned}
V_{hb}(t) = & \frac{\delta_b}{r} - \left(\frac{\kappa}{\kappa+\nu+r}\right)\left(\frac{x}{r}\right) \\
& + \left(\frac{\kappa}{\kappa+\nu+r}\right)\left(\frac{m_{hn}(t)[W_{lb}(t) - V_{lb}(t)]}{r}\right) \\
& + \left(\frac{\kappa}{\kappa+\nu+r}\right)\left(\frac{\dot{V}_{lb}(t)}{r}\right) + \left(\frac{\nu+r}{\kappa+\nu+r}\right)\left(\frac{\dot{V}_{hb}(t)}{r}\right).
\end{aligned}
\tag{19}
$$

Notice that this value is increasing in both $W_{lb}(t) - V_{lb}(t)$, the surplus accruing to a seller of a lemon, and $m_{hn}(t)$, the rate at which a seller meets buyers. In an equilibrium where all buyers offer the low price $\bar{p}_{lb}(t)$, sellers of lemons receive no surplus and (19) collapses to a time-invariant expression analogous to (17). On the other hand, when at least some buyers offer the high price $\bar{p}_{lg}(t)$ and all assets are traded, $W_{lb}(t) - V_{lb}(t)$ is strictly positive. Then, the value of holding a lemon is also increasing in the ease of finding a buyer. Given that a buyer's expected surplus from offering $\bar{p}_{lg}(t)$ is increasing in the value of owning a lemon, a buyer is more willing to offer the high price $\bar{p}_{lg}(t)$ when other buyers do likewise and when a seller meet buyers at a high rate. That is, buyers' price offers are strategic complements and high market tightness is conducive to all assets being traded. These features play prominent roles in the following results.

3.5 Stationary equilibria

Due to strategic complementarity in buyer's price offers, the environment gives rise to multiple stationary equilibria. Namely, when all assets are traded, the value of owning a lemon is above that justified by its dividend flow. This is due to lemons being bought at a price which reflects the dividend flow of a good asset. Due to the inflated value of lemons, an individual buyer is more willing to offer the high price, accepted by sellers of both types of assets, when all assets are traded. On the other hand, when only lemons are traded, the value of owning a lemon reflects its dividend flow, decreasing the expected surplus to a buyer from offering the high price. Thus, the following obtains.

Proposition 1. *For an open set of parameter values, there exists both a stationary equilibrium where only lemons are traded and a stationary equilibrium where all assets are traded. In the latter, only low-valuation owners are sellers.*

Proof. Suppose only low-valuation owners are willing to sell their asset, which will be verified to be the case for the parameter values considered below. Then, one obtains from (9) that the fractions of sellers having good assets in the different stationary equilibria satisfy

$$\tilde{\lambda}^a = \lambda \tag{20}$$

$$\tilde{\lambda}^o = \lambda \left(\frac{\kappa + \nu + m^o_{hn}}{\kappa + \nu + \lambda m^o_{hn}} \right), \tag{21}$$

where the superscript a denotes the stationary equilibrium where all assets are traded while the superscript o refers to the stationary equilibrium where only lemons are traded. Turning to value functions, using (7), (15) and (16) to solve for V_{ln} and V_{hn} yields

$$V^a_{hn} = \left(\frac{\nu + r}{\kappa + \nu + r + m^a_S} \right) \left(\frac{m^a_S(\lambda V_{hg} + (1-\lambda) V^a_{hb} - V_{lg})}{r} \right) \tag{22}$$

$$V^a_{ln} = \left(\frac{\nu}{\kappa + \nu + r + m^a_S} \right) \left(\frac{m^a_S(\lambda V_{hg} + (1-\lambda) V^a_{hb} - V_{lg})}{r} \right) \tag{23}$$

$$V^o_{hn} = \left(\frac{\nu + r}{\kappa + \nu + r + m^o_{lb}} \right) \left(\frac{m^o_{lb}(V^o_{hb} - V^o_{lb})}{r} \right) \tag{24}$$

$$V^o_{ln} = \left(\frac{\nu}{\kappa + \nu + r + m^o_{lb}} \right) \left(\frac{m^o_{lb}(V^o_{hb} - V^o_{lb})}{r} \right), \tag{25}$$

where $m_{lb} = \mu\gamma_{lb}/(\gamma_{hn} + \gamma_S)$. Next, consider owners of lemons. Given that $W^a_{lb} = V_{lg}$ and $W^o_{lb} = V^o_{lb}$, (1) and (2) imply

$$\begin{aligned} V^a_{hb} = & \left(\frac{r(\kappa + \nu + r + m^a_{hn})}{r(\kappa + \nu + r + m^a_{hn}) + m^a_{hn}\kappa} \right) \left(\frac{\delta_b}{r} \right) \\ & + \left(\frac{m^a_{hn}\kappa}{r(\kappa + \nu + r + m^a_{hn}) + m^a_{hn}\kappa} \right) \left(\frac{\delta_g}{r} \right) - \left(\frac{\kappa}{\kappa + \nu + r} \right) \left(\frac{x}{r} \right) \end{aligned} \tag{26}$$

$$\begin{aligned} V^a_{lb} = & \left(\frac{r(\kappa + \nu + r)}{r(\kappa + \nu + r) + m^a_{hn}(\kappa + r)} \right) \left(\frac{\delta_b}{r} \right) \\ & + \left(\frac{m^a_{hn}(\kappa + r)}{r(\kappa + \nu + r) + m^a_{hn}(\kappa + r)} \right) \left(\frac{\delta_g}{r} \right) - \left(\frac{\kappa + r}{\kappa + \nu + r} \right) \left(\frac{x}{r} \right) \end{aligned} \tag{27}$$

$$V^o_{hb} = \frac{\delta_b}{r} - \left(\frac{\kappa}{\kappa + \nu + r} \right) \left(\frac{x}{r} \right) \tag{28}$$

$$V^o_{lb} = \frac{\delta_b}{r} - \left(\frac{\kappa + r}{\kappa + \nu + r} \right) \left(\frac{x}{r} \right). \tag{29}$$

Note that $V^a_{lb} > V^o_{lb}$ and $V^a_{hb} > V^o_{hb}$. Moreover, in the equilibrium where only lemons are traded, high-valuation owners of a lemons prefers holding their asset to selling it as

$$\bar{p}^o_{hb} - \bar{p}^o_{lb} = \frac{x}{\kappa + \nu + r + m^o_{lb}} > 0. \tag{30}$$

Knowledge of the value functions allows one to find buyer's expected surplus from different price offers. Substituting for V_{hn} and V_{ln} in (11) and (12) yields

$$\Gamma(\bar{p}^a_{lg}) = \left(\frac{\kappa+\nu+r}{\kappa+\nu+r+m^a_S}\right)(\lambda V_{hg} + (1-\lambda)V^a_{hb} - V_{lg}) \tag{31}$$

$$\Gamma(\bar{p}^a_{lb}) = (1-\lambda)\left[V^a_{hb} - V^a_{lb} - \left(\frac{m^a_S}{\kappa+\nu+r}\right)\Gamma(\bar{p}^a_{lg})\right] \tag{32}$$

$$\Gamma(\bar{p}^o_{lg}) = \tilde{\lambda}^o V_{hg} + (1-\tilde{\lambda}^o)V^o_{hb} - V_{lg} - \left(\frac{m^o_{lb}}{\kappa+\nu+r}\right)\Gamma(\bar{p}^o_{lb}) \tag{33}$$

$$\Gamma(\bar{p}^o_{lb}) = (1-\tilde{\lambda}^o)\left(\frac{\kappa+\nu+r}{\kappa+\nu+r+m^o_{lb}}\right)(V^o_{hb} - V^o_{lb}). \tag{34}$$

The existence of the equilibrium where all assets are traded requires

$$\Gamma(\bar{p}^a_{lg}) \geq \Gamma(\bar{p}^a_{lb}). \tag{35}$$

Inspecting (31) and (32) leads one to conjecture that there exists $\bar{\lambda}^a \in (0,1)$ such that (35) is satisfied for all $\lambda \geq \bar{\lambda}^a$. To prove this conjecture, first note that $\Gamma(\bar{p}^a_{lg})$ is linearly increasing in λ as $V_{hg} > V^a_{hb}$ and strictly positive for $\lambda = 1$ since $V_{hg} > V_{lg}$. On the other hand, $\Gamma(\bar{p}^a_{lb})$ as a function of λ is a parabola opening upwards as $V_{hg} > V^a_{hb}$. Moreover, $V_{lg} > V^a_{lb}$ implies that $\Gamma(\bar{p}^a_{lb}) > \Gamma(\bar{p}^a_{lg})$ at $\lambda = 0$. Thus, given that $\Gamma(\bar{p}^a_{lb})$ is equal to zero at $\lambda = 1$, there exists a unique $\bar{\lambda}^a \in (0,1)$ at which $\Gamma(\bar{p}^a_{lg}) = \Gamma(\bar{p}^a_{lb})$ and $\Gamma(\bar{p}^a_{lg}) > \Gamma(\bar{p}^a_{lb})$ for all $\lambda \in (\bar{\lambda}^a, 1]$. Also, note that $\Gamma(\bar{p}^a_{lg})$ evaluated at $\bar{\lambda}^a$ is strictly positive as otherwise $\Gamma(\bar{p}^a_{lb})$ at $\bar{\lambda}^a$ would be strictly positive, constituting a contradiction. Thus, buyers prefer offering $\bar{p}^a_{lg}$ also to offering a price rejected by all sellers. Turning to the equilibrium where only lemons are traded, offering $\bar{p}^o_{lb}$ is optimal for buyers when

$$\Gamma(\bar{p}^o_{lb}) \geq \Gamma(\bar{p}^o_{lg}). \tag{36}$$

Using (33) and (34) and the fact that $\tilde{\lambda}$ is an increasing function of λ, one can establish by straightforward calculations that $\Gamma(\bar{p}^o_{lg})$ is increasing in λ and $\Gamma(\bar{p}^o_{lb})$ is decreasing in λ. Moreover, $\Gamma(\bar{p}^o_{lb}) > \Gamma(\bar{p}^o_{lg})$ at $\lambda = 0$ and $\Gamma(\bar{p}^o_{lg}) > \Gamma(\bar{p}^o_{lb})$ at $\lambda = 1$. Therefore, there exists $\bar{\lambda}^o \in (0,1)$ such that (36) is satisfied if and only if $\lambda \leq \bar{\lambda}^o$. Clearly, $\Gamma(\bar{p}^o_{lb})$ evaluated at $\bar{\lambda}^o$ is strictly positive, ensuring that buyers prefer offering p^o_l to not trading. Finally, to establish that there is an open set of parameter values for which both stationary equilibria exist, one needs to show that $\bar{\lambda}^o > \bar{\lambda}^a$. This inequality holds if $\Gamma(\bar{p}^o_{lb}) - \Gamma(\bar{p}^o_{lg})$ evaluated at $\bar{\lambda}^a$ is strictly positive. From (11) and (12), one obtains

$$\Gamma(\bar{p}^a_{lg}) - \Gamma(\bar{p}^a_{lb}) = \lambda[V_{hg} - (V^a_{hn} - V^a_{ln})] - V_{lg} + (1-\lambda)V^a_{lb} \tag{37}$$

$$\Gamma(\bar{p}^o_{lb}) - \Gamma(\bar{p}^o_{lg}) = V_{lg} - (1-\tilde{\lambda}^o)V^o_{lb} - \tilde{\lambda}^o[V_{hg} - (V^o_{hn} - V^o_{ln})]. \tag{38}$$

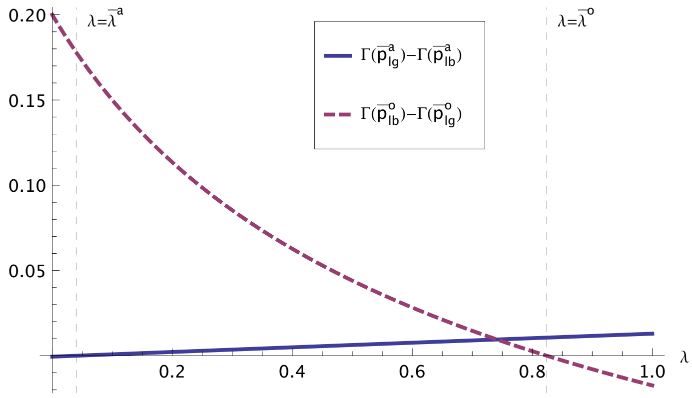

Figure 2: Multiplicity of stationary equilibria for $\delta_g = 1$, $\delta_b = 0.99$, $x = 0.61$, $\kappa = 10$, $\nu = 25$, $\mu = 75$, $A = 1$ and $r = 0.05$. For a fraction of good assets λ between the two dashed lines $\lambda = \bar{\lambda}^a$ and $\lambda = \bar{\lambda}^o$ both a stationary equilibrium where all assets are traded and a stationary equilibrium where only lemons are traded exist. The parameter values are such that $\bar{p}_{hg} > \bar{p}_{lg}$ for all $\lambda \in [0,1]$, implying that only low-valuation owners are sellers.

Let $\kappa = \nu = r$ and $A = 1$. This implies that $m_S^a = \mu/2$ and $m_{lb}^o = (1-\tilde{\lambda}^o)\mu/2$. Also, let $\kappa \to \infty$. Then, from (21), $\tilde{\lambda}^o \to \lambda$. To ensure that high-valuation owners of lemons prefer holding their asset to selling it in the equilibrium where all assets are traded it is sufficient to impose the parameter restrictions $\delta_b > \delta_g - x/3$ as

$$\lim_{\kappa\to\infty} r(\bar{p}_{hb}^a - \bar{p}_{lg}^a) = \delta_b - \left(\delta_g - \frac{x}{3}\right) \tag{39}$$

when $\kappa = \nu = r$. Moreover, (22), (23), (24) and (25) imply that $V_{hn}^a - V_{ln}^a \to V_{hn}^o - V_{ln}^o$. Then, using (37) and (38), $\Gamma(\bar{p}_{lb}^o) - \Gamma(\bar{p}_{lg}^o)$ evaluated at $\bar{\lambda}^a$ is equal to

$$(1-\bar{\lambda}^a)(V_{lb}^a - V_{lb}^o) > 0. \tag{40}$$

By continuity, this remains true in the neighborhood of the parameter values considered here. □

Expression (40) reveals the source of the multiplicity. The value of owning a lemon is higher in the equilibrium where all assets are traded than in the equilibrium where only lemons are traded, i.e. $V_{lb}^a > V_{lb}^o$. Thus, the possibility of obtaining a lemon deters buyers less from offering the high price $\bar{p}_{lg}$ when all assets are traded than when only lemons are traded. Figure 2 illustrates the multiplicity of stationary equilibria. When all assets are traded, an individual buyer obtains a higher expected surplus from offering the high price $\bar{p}_{lg}^a$ than from offering the low price $\bar{p}_{lb}^a$ when the fraction of good assets λ is sufficiently high.

That is, $\Gamma(\bar{p}^a_{lg}) > \Gamma(\bar{p}^a_{lb})$ for $\lambda > \bar{\lambda}^a$. On the other hand, when only lemons are traded, an individual buyer will find it optimal to offer the low price $\bar{p}^o_{lb}$ rather than the high price $\bar{p}^o_{lg}$ unless the fraction of good assets exceeds the threshold $\bar{\lambda}^o$. As $\bar{\lambda}^o > \bar{\lambda}^a$, both a stationary equilibrium where all assets are traded and a stationary equilibrium where only lemons are traded exist.

As a prelude to showing how a permanent market freeze can arise in this environment, let me next analyze the determinants of the rate at which buyers and sellers meet in the different stationary equilibria. From (9) and (10), one finds that in any stationary equilibrium

$$\gamma_{hn} - \gamma_l = \frac{\nu - \kappa A}{\kappa + \nu}, \tag{41}$$

where $\gamma_l = \gamma_{lb} + \gamma_{lg}$. Notice that the measure of low-valuation owners relative to that of buyers is increasing in κ and decreasing in ν. This is intuitive as the higher is the rate at which agents transit to the state of low valuation, the more low-valuation owners and the less buyers there are. On the other hand, increasing the rate at which agents transit to the state of high valuation leads to a larger pool of buyers and a smaller pool of low-valuation owners. Moreover, (41) reveals that whenever $\nu > \kappa A$, there are more buyers than low-valuation owners in a stationary equilibrium. Next, let me investigate the rate at which a seller meet buyers, m_{hn}, across stationary equilibria.

Proposition 2. *For $\nu > \kappa A$, the rate at which a seller meets buyers m_{hn} is strictly lower in the stationary equilibrium where only lemons are traded than in the stationary equilibrium where all assets are traded and only low-valuation owners are sellers.*

Proof. Consider parameter values for which only low-valuation owners are sellers, i.e. $\gamma_S = \gamma_{lb} + \gamma_{lg}$. Dividing (41) by γ_l and rearranging yields

$$\frac{\gamma_{hn}}{\gamma_l} = 1 + \frac{\nu - \kappa A}{\kappa + \nu}\frac{1}{\gamma_l}. \tag{42}$$

For $\nu > \kappa A$, γ_{hn}/γ_l is decreasing in γ_l. Hence, to prove that $\gamma_{hn}/(\gamma_{hn}+\gamma_l)$, which is increasing in γ_{hn}/γ_l, is lower in the stationary equilibrium where only lemons are traded, it is sufficient to show that γ_l is higher in the stationary equilibrium where only lemons are traded. Suppose not. That is, $\gamma^a_l > \gamma^o_l$. Then, it follows from (42) that

$$\frac{\gamma^o_{hn}}{\gamma^o_l} > \frac{\gamma^a_{hn}}{\gamma^a_l}. \tag{43}$$

This implies that

$$\frac{\gamma^o_l}{\gamma^o_{hn} + \gamma^o_l} < \frac{\gamma^a_l}{\gamma^a_{hn} + \gamma^a_l}. \tag{44}$$

Given that rearranging (10) yields

$$\gamma^a_{hn}\left(\kappa+\nu+\frac{\mu\gamma^a_l}{\gamma^a_{hn}+\gamma^a_l}\right)=\nu \tag{45}$$

$$\gamma^o_{hn}\left(\kappa+\nu+\frac{\mu(1-\tilde{\lambda}^o)\gamma^o_l}{\gamma^o_{hn}+\gamma^o_l}\right)=\nu, \tag{46}$$

it follows that $\gamma^o_{hn} > \gamma^a_{hn}$. However, (41) requires that $\gamma^o_{hn}-\gamma^a_{hn}=\gamma^o_l-\gamma^a_l$. Thus, $\gamma^o_l > \gamma^a_l$, which constitutes a contradiction. Therefore, $m^o_{hn} < m^a_{hn}$ for parameter value for which only low-valuation agents are sellers and satisfying $\nu > \kappa A$. □

The intuition for why, under the parameter restrictions established above, the rate at which a seller meets buyers is lower when only lemons are traded can be understood as follows. Note that when parameter values satisfy the conditions specified above, there are more buyers than low-valuation sellers in the stationary equilibrium where all assets are traded. When the market transits to the stationary equilibrium where only lemons are traded the measures of buyers and sellers increase. This is due to the fact that not in all encounters between buyers and sellers trade takes place . When there are initially more buyers than sellers, equally large absolute increases in the measures of buyers and sellers result in a larger relative increase in the measure of sellers than the measure of buyers.

Figure 3 shows the measures of agents with different statuses in the two stationary equilibria. When trade of good assets ceases, sellers of good assets accumulate in the market. Similarly, if the rate at which a seller meets buyers m_{hn} falls, there is an increase in the measure of low-valuation owners of lemons.

4 Permanent market freeze

I will show how a permanent market freeze can arise in this environment in three steps. First, I prove that, for some parameter values, multiplicity of stationary equilibria implies the existence of a transition path from the equilibrium where all assets are traded to the equilibrium where only lemons are traded. That is, there exists an equilibrium path converging to the stationary equilibrium where only lemons are traded from initial conditions determined by the stationary equilibrium where all assets are traded. Second, I show that high-valuation owners of lemons may switch from holding to selling if trade of all assets resumes. Furthermore, I prove the non-existence of a transition path along which all assets are traded and high-valuation owners of lemons switch from selling to holding. Thus, in the third step, I analyze a transition path along which all assets are traded and high-valuation owners sell their asset. I show that such a

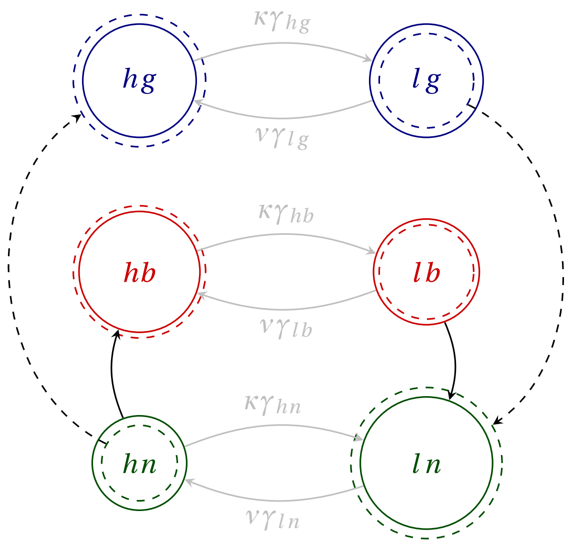

Figure 3: Evolution of measures of agents in the different stationary equilibria. The black solid lines indicate flows due to trade when only lemons are traded whereas the dashed lines indicate additional flows due to trade when all assets are traded. Similarly, the dashed circles represent the measures of agents when all assets are traded while the solid circles those when only lemons are traded.

transition path does not always constitute an equilibrium as the average quality of assets in the market falls when additional holders of lemons become sellers. Consequently, buyers may not be willing to switch to offering high prices, which would resume the trade of all assets.

To explore the possibility of a self-fulfilling market freeze, consider a point in time at which the market is in the stationary equilibrium where all assets are traded. Suppose that buyers' expectations about future trading opportunities change. More specially, buyers begin to expect that as from now only lemons will be traded. If there exists a transition path from the equilibrium where all assets are traded to the equilibrium where only lemons are traded, the buyers' changed expectations will be self-fulfilled. The next proposition specifies conditions under which multiplicity of stationary equilibria implies that such a transition path exists.

Proposition 3. *Consider parameter values satisfying $\nu \geq \kappa A$ and for which only low-valuation owners are willing to sell their asset in the stationary equilibrium where all assets are traded. Then, the existence of the stationary equilibrium where only lemons are traded implies that there exists a transition path along which only lemons are traded originating from initial conditions determined by the stationary equilibrium where all assets are traded.*

Proof. The stationary equilibrium where only lemons are traded exists when

$$\Gamma(\bar{p}_{lb}^{o}) - \Gamma(\bar{p}_{lg}^{o}) = V_{lg} - V_{lb}^{o} - \tilde{\lambda}^{o}[V_{hg} - V_{lb}^{o} - (V_{hn}^{o} - V_{ln}^{o})] \geq 0. \tag{47}$$

The existence of a transition path along which only lemons are traded, on the other hand, requires that[15]

$$\Gamma(\bar{p}_{lb}(t)) - \Gamma(\bar{p}_{lg}(t)) = V_{lg} - V_{lb}^{o} - \tilde{\lambda}(t)[V_{hg} - V_{lb}^{o} - (V_{hn}(t) - V_{ln}(t))] \geq 0 \tag{48}$$

holds for all $t \geq 0$. Thus, if $\tilde{\lambda}(t) \leq \tilde{\lambda}^{o}$ and $V_{hn}(t) - V_{ln}(t) \geq V_{hn}^{o} - V_{ln}^{o}$ for all $t \geq 0$, then (47) implies (48). One can prove that these two inequalities are satisfied by analyzing the evolution of measures of agents. When only lemons are traded, one obtains from (9) and (10) that[16]

$$\dot{\gamma}_{lg} = \kappa\lambda A - (\kappa+\nu)\gamma_{lg} \tag{49}$$

$$\dot{\gamma}_{lb} = \kappa(1-\lambda)A - (\kappa+\nu)\gamma_{lb} - \frac{\mu\gamma_{lb}\gamma_{hn}}{\gamma_{hn}+\gamma_{l}} \tag{50}$$

$$\dot{\gamma}_{hn} = \nu - (\kappa+\nu)\gamma_{hn} - \frac{\mu\gamma_{lb}\gamma_{hn}}{\gamma_{hn}+\gamma_{l}}. \tag{51}$$

First note that combining these three laws of motion yields

$$\dot{\gamma}_{hn} - (\dot{\gamma}_{lb} + \dot{\gamma}_{lg}) = \nu - \kappa A - (\kappa+\nu)[\gamma_{hn} - (\gamma_{lb}+\gamma_{lg})]. \tag{52}$$

Given that the initial conditions are determined by the stationary equilibrium where all assets are traded, it follows from (41) that $\gamma_{hn} - \gamma_{l} = (\nu - \kappa A)/(\kappa+\nu)$ at time 0. Thus, (52) implies that $\gamma_{hn} - \gamma_{l} = (\nu - \kappa A)/(\kappa+\nu)$ for all $t \geq 0$. Using this result to substitute for γ_{hn} in (50) yields

$$\dot{\gamma}_{lb} = \kappa(1-\lambda)A - (\kappa+\nu)\gamma_{lb} - \frac{\mu\gamma_{lb}(\gamma_{lb}+\gamma_{lg}+\alpha)}{2(\gamma_{lb}+\gamma_{lg})+\alpha}, \tag{53}$$

where $\alpha = (\nu - \kappa A)/(\kappa+\nu)$. Note that $\dot{\gamma}_{lb}$ is decreasing in γ_{lb}. Moreover, implicitly differentiating $\dot{\gamma}_{lb} = 0$ with respect to γ_{lg} yields

$$\frac{\partial\gamma_{lb}}{\partial\gamma_{lg}} = \frac{\alpha\mu\gamma_{lb}}{(\kappa+\nu)(2(\gamma_{lb}+\gamma_{lg})+\alpha)^{2} + \mu[2(\gamma_{lb}+\gamma_{lg})(\gamma_{lb}+\gamma_{lg}+\alpha)+\alpha(\alpha+\gamma_{lg})]}, \tag{54}$$

which is a positive expression. Similarly, analyzing (49) reveals that $\dot{\gamma}_{lg}$ is decreasing in γ_{lg}. Thus, given that $\gamma_{lg}^{a} < \gamma_{lg}^{o}$ and $\dot{\gamma}_{lb}$ evaluated at $(\gamma_{lb}^{a}, \gamma_{lb}^{a})$ is equal

[15] Given that only lemons are traded along the transition path, V_{lb}^{o} is time-invariant and equal to its steady state value.

[16] In the rest of the proof, the dependence of the masses of agents and of the value functions on time is suppressed for conciseness.

to 0, a phase diagram analysis, illustrated in Figure 4, reveals that both $\dot{\gamma}_{lb} \geq 0$ and $\dot{\gamma}_{lg} \geq 0$ along the transition path under consideration. Consequently, γ_l is increasing along the transition path. One can utilize these insights to prove that on the transition path $\tilde{\lambda} \leq \tilde{\lambda}^o$. Using the definition of $\tilde{\lambda}$ along with (49) and (50) yields

$$\dot{\tilde{\lambda}} = \frac{1}{\gamma_l}\left[\frac{\mu\tilde{\lambda}(1-\tilde{\lambda})\gamma_l(\gamma_l+\alpha)}{2\gamma_l+\alpha} - \kappa(\tilde{\lambda}-\lambda)A\right]. \tag{55}$$

Let $f(\gamma_l,\tilde{\lambda}) := \dot{\tilde{\lambda}}$. Note that $f(\gamma_l^o,\tilde{\lambda}^o) = 0$. Moreover, given that the first term in square brackets in (55) is increasing in γ_l, $f(\gamma_l,\tilde{\lambda}^o) < 0$ for all $\gamma_l < \gamma_l^o$. Also, recall that γ_l is increasing along the transition path. Then, it follows that $\tilde{\lambda} \leq \tilde{\lambda}^o$ since $\tilde{\lambda}^o > \tilde{\lambda}^a$. Finally, one can prove that $V_{hn} - V_{ln} \geq V_{hn}^o - V_{ln}^o$ along the transition path by investigating the evolution of m_{lb}. Employing the definition of m_{lb}, (50) and (51), one obtains

$$\dot{m}_{lb} = \frac{1}{2\gamma_{hn}-\alpha}\left[\mu\kappa(1-\lambda)A - 2\nu m_{lb} - \gamma_{hn} m_{lb}(\mu - 2m_{lb})\right]. \tag{56}$$

Given that $\mu - 2m_{lb} = \mu(1-2\gamma_{lb}/(2\gamma_l+\alpha))$, the term in square brackets is decreasing in γ_{hn}. Let $g(\gamma_{hn},m_{lb}) := \dot{m}_{lb}$. As $g(\gamma_{hn}^o,m_{lb}^o) = 0$ and $2\gamma_{hn}-\alpha = \gamma_{hn}+\gamma_l > 0$, one can see that $g(\gamma_{hn},m_{lb}^o) > 0$ for all $\gamma_{hn} < \gamma_{hn}^o$. Moreover, $\gamma_{hn}^o > \gamma_{hn}^a$ implies that $m_{lb}^o < m_{lb}^a$ since $\gamma_{hn}^o = \nu/(\kappa+\nu+m_{lb}^o)$. Thus, $m_{lb} \geq m_{lb}^o$ along the transition path. To prove that this implies $V_{hn} - V_{ln} \geq V_{hn}^o - V_{ln}^o$, suppose otherwise, i.e. that there exists t at which $V_{hn} - V_{ln} < V_{hn}^o - V_{ln}^o$. Then, it follows that $\dot{V}_{hn} - \dot{V}_{ln} > 0$ for some (V_{hn}, V_{ln}) satisfying $V_{hn} - V_{ln} < V_{hn}^o - V_{ln}^o$. Using (15) and (16) to solve for $\dot{V}_{hn} - \dot{V}_{ln}$ along the transition path yields

$$\dot{V}_{hn} - \dot{V}_{ln} = (\kappa+\nu+r+m_{lb})(V_{hn}-V_{ln}) - m_{lb}(V_{hb}-V_{lb}). \tag{57}$$

Thus, $\dot{V}_{hn} - \dot{V}_{ln} > 0$ when

$$V_{hn} - V_{ln} > \frac{m_{lb}(V_{hb}-V_{lb})}{\kappa+\nu+r+m_{lb}}. \tag{58}$$

However, given that

$$V_{hn}^o - V_{ln}^o = \frac{m_{lb}^o(V_{hb}-V_{lb})}{\kappa+\nu+r+m_{lb}^o} \tag{59}$$

and $m_{lb} \geq m_{lb}^o$, it follows that $\dot{V}_{hn} - \dot{V}_{ln} < 0$ when $V_{hn} - V_{ln} < V_{hn}^o - V_{ln}^o$. This contradiction completes the proof. □

Intuitively, there are two reasons for why the existence of the stationary equilibrium where only lemons are traded guarantees that a transition path to that

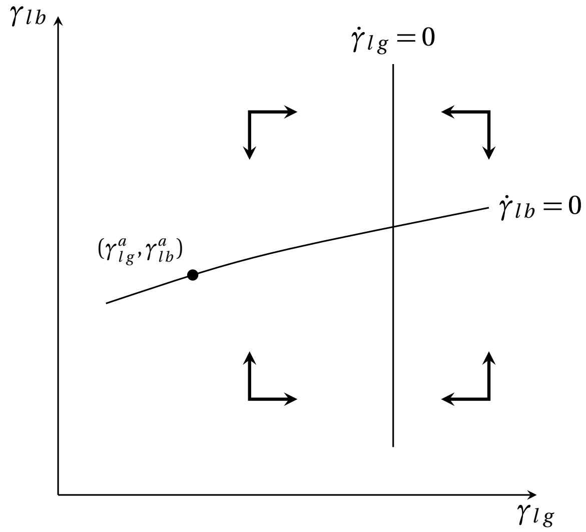

Figure 4: Phase diagram for γ_{lg} and γ_{lb} along the transition path where only lemons are traded.

equilibrium exists. First, along the transition path the fraction of sellers offering lemons is higher than in the limiting stationary equilibrium, increasing the probability that a buyer's offer is accepted. Second, sellers of lemons have a lower reservation price on the transition path than in the limiting stationary equilibrium. Thus, along the transition path a buyer captures a larger surplus from a successful trade than in the limiting stationary equilibrium. Due to the combination of these two forces, it is optimal for buyers to offer the low price throughout the transition path, ensuring the sustainability of a self-fulfilling market freeze.

To assess whether self-fulfilling expectations can also support recovery from a market freeze, consider the reverse transition path along with all assets are traded and with initial conditions determined by the stationary equilibrium where only lemons are traded. When only lemons are traded, sellers of good assets accumulate in the market, resulting in an increase in the fraction of sellers offering good assets. This increases a buyer's expected surplus from offering the high price $\bar{p}_{lg}$ when other buyers do likewise. Moreover, the market can inherit a high rate at which a buyer meets sellers m_S from the equilibrium where only lemons are traded,[17] further increasing the value of being a buyer. As a result, high-valuation owners of lemons may wish to sell their asset and enter the pool of buyers. It is optimal for a high-valuation owner of a lemon to sell

[17]Given that m_S is inversely related to m_{hn}, $m^o_S > m^a_S$ when the conditions specified in Proposition 2 are satisfied.

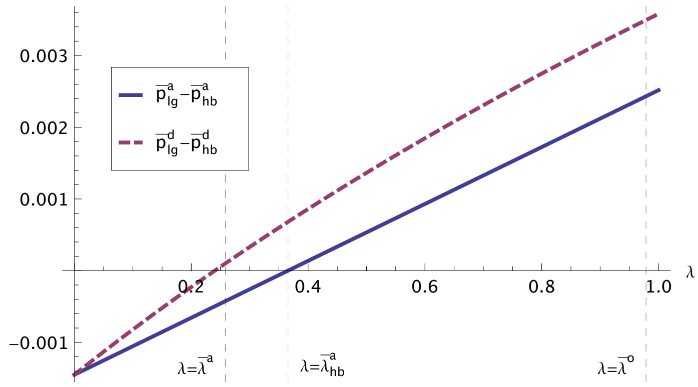

Figure 5: Reservation prices at time 0 on the transition path along which all assets are traded and only low-valuation owners sell their asset (d) along with those the limiting stationary equilibrium (a) for $\delta_g = 1$, $\delta_b = 0.9$, $x = 0.61$, $\kappa = 10$, $\nu = 25$, $\mu = 75$, $A = 1$ and $r = 0.05$. For a fraction of good assets between $\lambda = \bar{\lambda}^a$ and $\lambda = \bar{\lambda}^a_{hb}$ both stationary equilibria exist.

their asset when their reservation price $\bar{p}_{hb}$ is below the price offered by buyers. Figure 5 shows that there exists parameter values for which multiple stationary equilibria exist and high-valuation owners of lemons would be willing to sell their asset if trade of all assets resumed.[18] This can be seen from $\bar{p}^d_{lg} - \bar{p}^d_{hb} > 0$ for $\lambda \in [\bar{\lambda}^a, \bar{\lambda}^a_{hb}]$.[19] Thus, there are parameter values for which the reverse transition path along which only low-valuation owners are willing to sell their asset does not exist. This finding is formalized in the following proposition.

Proposition 4. *For an open set of parameters satisfying $\kappa > \nu A$, high-valuation owners of lemons are willing to sell their asset on the transition path along which all assets are traded but not in the limiting stationary equilibrium.*

Proof. It is to be shown that $\bar{p}_{lg}(t) - \bar{p}_{hg}(t) > 0$ on the transition path along which all assets are traded but only low-valuation owners are sellers. At the same time, $\bar{p}^a_{lg} - \bar{p}^a_{hg} \leq 0$. Consider parameter values for which $\bar{p}^a_{lg} - \bar{p}^a_{hg} = 0$. From the proof of Proposition 1, this equality is satisfied when the parameter x is cho-

[18] In Figure 5, both stationary equilibria exist only for $\lambda \in [\bar{\lambda}^a, \bar{\lambda}^a_{hb}$ as for $\lambda > \bar{\lambda}^a_{hb}$ a high-valuation owner of a lemon would sell their asset.

[19] The superscript d denotes time 0 on the transition path along which only low-valuation owners sell their asset.

sen appropriately. Given that[20]

$$\bar{p}_{lg} - \bar{p}_{hg} = V_{lg} - V_{hb} + V_{hn} - V_{ln}, \tag{60}$$

it is first shown that V_{hb} is below its steady state value along the transition path. From (13) and (14), one obtains

$$\dot{V}_{hb} = (\kappa + r)V_{hb} - \kappa V_{lb} - \delta_b \tag{61}$$

$$\dot{V}_{lb} = (\nu + r + m_{hn})V_{lb} - \nu V_{hb} - (\delta_b - x) - m_{hn} V_{lg}. \tag{62}$$

In order to be able to analyze this system of differential equations, the evolution of m_{hn} is first characterized. Proceeding as in proving Proposition 3, one finds that

$$m_{hn} = \frac{\mu \gamma_{hn}}{2\gamma_{hn} - \alpha}, \tag{63}$$

where $\alpha = (\nu - \kappa A)/(\kappa + \nu)$. Thus, if $\dot{\gamma}_{hn} \leq 0$, then $\dot{m}_{hn} \geq 0$. Using $\gamma_{hn} - \gamma_l = \alpha$ to substitute for γ_l in (10) yields

$$\dot{\gamma}_{hn} = \nu - (\kappa + \nu)\gamma_{hn} - \frac{\mu \gamma_{hn}(\gamma_{hn} - \alpha)}{2\gamma_{hn} - \alpha}. \tag{64}$$

Given that $\dot{\gamma}_{hn}$ is decreasing in γ_{hn} and $\gamma^o_{hn} > \gamma^a_{hn}$, as established in the proof of Proposition 2, it follows that $\dot{\gamma}_{hn} \leq 0$. Thus, $\dot{m}_{hn} \geq 0$ along the transition path. Also note that this implies $\dot{m}_S \leq 0$. Using the fact that m_{hn} is increasing along the transition path, one obtains the phase diagram for V_{hb} and V_{lb} illustrated in Figure 6. The point $(\bar{V}_{hb}, \bar{V}_{lb})$ denotes the pair of values for which $\dot{V}_{hb} = 0$ and $\dot{V}_{lb} = 0$ at time t. Given that

$$\frac{\mathrm{d}\bar{V}_{lb}}{\mathrm{d}m_{hn}} = \frac{(\delta_g - \delta_b)(\kappa + r)(\kappa + \nu + r)}{[r(\kappa + \nu + r + m_{hn}) + m_{hn}\kappa]^2} > 0, \tag{65}$$

it follows that $V^a_{hb} > \bar{V}_{hb}$ and $V^a_{lb} > \bar{V}_{lb}$. Moreover, one can prove by contradiction that $\bar{V}_{hb} < V_{hb} < V^a_{hb}$ and $\bar{V}_{lb} < V_{lb} < V^a_{lb}$ along the transition path. Turning to $V_{hn} - V_{ln}$, combining (15) and (16) yields

$$\dot{V}_{hn} - \dot{V}_{ln} = (\kappa + \nu + r + m_S)(V_{hn} - V_{ln}) - m_S[\tilde{\lambda} V_{hg} + (1 - \tilde{\lambda})V_{hb} - V_{lg}]. \tag{66}$$

To prove that $\bar{p}_{lg} - \bar{p}_{hg} > 0$ along the transition path suppose otherwise. That is,

$$V_{hn} - V_{ln} \leq V_{hb} - V_{lg}. \tag{67}$$

[20] In the rest of the proof, the dependence of the value functions and of measures of agents on time is suppressed for conciseness.

Given that $V^a_{hn} - V^a_{ln} = V^a_{hb} - V_{lg}$ and $V_{hb} < V^a_{hb}$, (67) implies that there exists $V_{hn} - V_{ln}$ for which $\dot{V}_{hn} - \dot{V}_{ln} > 0$. From (66), this is equivalent to

$$V_{hn} - V_{ln} > \frac{m_S}{\kappa + \nu + r + m_S}[\tilde{\lambda} V_{hg} + (1-\tilde{\lambda})V_{hb} - V_{lg}]. \tag{68}$$

Combining this with (67) yields,

$$V_{hb} - V_{lg} > \frac{m_S}{\kappa + \nu + r + m_S}[\tilde{\lambda} V_{hg} + (1-\tilde{\lambda})V_{hb} - V_{lg}]. \tag{69}$$

Note that this expression holds at equality in the stationary equilibrium as then $\dot{V}_{hn} - \dot{V}_{ln} = 0$ and $V^a_{hn} - V^a_{ln} = V^a_{hb} - V_{lg}$. By rearranging one obtains

$$V_{hb} > \left(\frac{m_S\tilde{\lambda}}{\kappa + \nu + r + m_S\tilde{\lambda}}\right) V_{hg} + \left(\frac{\kappa + \nu + r}{\kappa + \nu + r + m_S\tilde{\lambda}}\right) V_{hg}. \tag{70}$$

The right-hand side of this expression is increasing in $m_S\tilde{\lambda}$. Thus, if $m_S\tilde{\lambda} \geq m^a_S\lambda$ along the transition path, $V_{hb} < V^a_{hb}$ violates (70). It has already been established that $\dot{m}_S \leq 0$, implying that $m_S \geq m^a_S$. To show that $\tilde{\lambda} > \lambda$ along the transition path, consider γ_{lg} and γ_{hg}. From (9), one obtains

$$\dot{\gamma}_{lg} = \kappa\lambda A - (\kappa + \nu)\gamma_{lg} - \gamma_{lg} m_{hn} \tag{71}$$

$$\dot{\gamma}_{lb} = \kappa(1-\lambda)A - (\kappa + \nu)\gamma_{lb} - \gamma_{lb} m_{hn}. \tag{72}$$

Differentiating the definition of $\tilde{\lambda}$ with respect to time yields

$$\dot{\tilde{\lambda}} = \frac{\dot{\gamma}_{lg}\gamma_l - (\dot{\gamma}_{lg} + \dot{\gamma}_{lb})\gamma_{lg}}{\gamma_l^2} \tag{73}$$

Substituting for $\dot{\gamma}_{lg}$ and $\dot{\gamma}_{lb}$, one obtains

$$\dot{\tilde{\lambda}} = \left(\frac{\kappa A}{\gamma_l}\right)(\lambda - \tilde{\lambda}). \tag{74}$$

Note that $\dot{\tilde{\lambda}} > 0$ is equivalent to $\tilde{\lambda} > \lambda$. Given that $\tilde{\lambda}^o > \lambda$, it has been established that $\tilde{\lambda} > \lambda$ along the transition path. Thus, (70) constitutes a contradiction. It has been established that $\bar{p}_{lg}(t) - \bar{p}_{hg}(t) > 0$ along the transition path. By continuity,[21] this remains true for parameter values for which $\bar{p}^a_{lg} - \bar{p}^a_{hg} > 0$ and for t sufficiently small. □

The above proposition shows, for some parameter values, the nonexistence of a transition path throughout which only low-valuation owners are sellers. Moreover, the proposition shows that if trade of all assets resumed, high-valuation owners of lemons would be willing to sell their asset. Thus, it is natural

[21] By Theorem on page 395 in Hirsch et al. (2004), $\bar{p}_{lg}(t) - \bar{p}_{hg}(t)$ depends continuously on the parameters of the model.

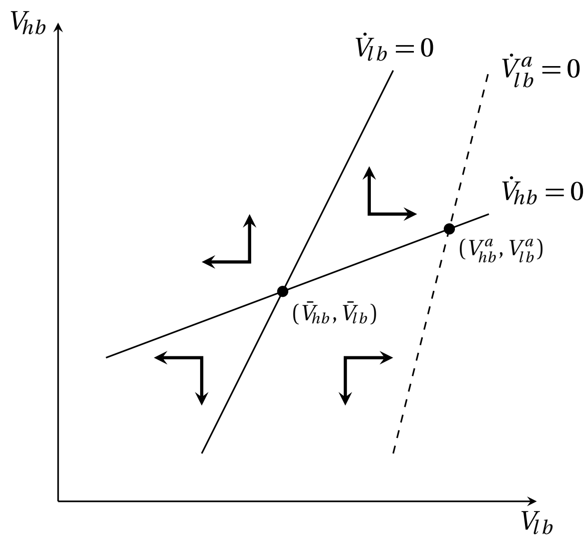

Figure 6: Phase diagram for V_{hb} and V_{lb} along the transition path where all assets are traded but only low-valuation owners are sellers.

to investigate whether there exists a transition path along which high-valuation owners of lemons are sellers up to some time $\bar{t}$, after which only low-valuation owners sell their asset but all assets continue to be traded. If such a transition path does not exist for parameter values for which the above proposition holds, then the stationary equilibrium where all asset are traded and only low-valuation owners are sellers cannot be reached from the initial conditions determined by the market freeze. This is established in the following proposition.

Proposition 5. *For an open set of parameter values satisfying $\kappa > \nu A$, neither a transition path along which all assets are traded and high-valuation owners of lemons switch from sellers to holders nor a transition path along which all assets are traded and only low-valuation owners are sellers exists.*

Proof. Considering the first part of the proposition, it is to be shown that there does not exist $\bar{t} \geq 0$ such that $\bar{p}_{lg}(t) - \bar{p}_{hg}(t) > 0$ for all $t < \bar{t}$ and $\bar{p}_{lg}(t) - \bar{p}_{hg}(t) \leq 0$ for all $t \geq \bar{t}$. Suppose otherwise. Then, $\bar{p}_{lg}(\bar{t}) - \bar{p}_{hg}(\bar{t}) = 0$. That is, at time $\bar{t}$ high-valuation owners of lemons are indifferent between selling and holding. As in the proof of Proposition 4, consider parameter values for which $\bar{p}_{lg}^a - \bar{p}_{hg}^a = 0$. Then, if $\gamma_{hn}(\bar{t}) > \gamma_{hn}^a$ and $\tilde{\lambda}(\bar{t}) \geq \lambda$, one can follow the same steps as in the proof of Proposition 4 to establish a contradiction. To find $\gamma_{hn}(\bar{t})$ and $\tilde{\lambda}(\bar{t})$, consider the evolution of measures of agents when all assets are traded and also high-valuation owners of lemons are sellers. When $\gamma_S = \gamma_l + \gamma_{hb}$, (9) implies that

$$\dot{\gamma}_{lg} = \kappa\lambda A - (\kappa+\nu)\gamma_{lg} - \frac{\mu\gamma_{hn}\gamma_{lg}}{\gamma_{hn}+\gamma_l+\gamma_{hb}} \tag{75}$$

$$\dot{\gamma}_{lb} = \kappa(1-\lambda)A - (\kappa+\nu)\gamma_{lb} - \frac{\mu\gamma_{hn}\gamma_{lg}}{\gamma_{hn}+\gamma_l+\gamma_{hb}}. \tag{76}$$

Thus, substituting for γ_{lg} and γ_{lb} in the expression for $\dot{\tilde{\lambda}}$ as in the proof above yields

$$\dot{\tilde{\lambda}} = \left(\frac{\kappa A}{\gamma_l}\right)(\lambda - \tilde{\lambda}). \tag{77}$$

Given that $\tilde{\lambda}^o > \lambda$, this implies that $\tilde{\lambda} \geq \lambda$ throughout the transition path along which also high-valuation owners of lemons are sellers. Turning to γ_{hn} and using $\gamma_l = \gamma_{hn} - \alpha$ and $\gamma_S = (1-\lambda)A - \gamma_{lg}$, (10) becomes

$$\dot{\gamma}_{hn} = \nu - (\kappa+\nu)\gamma_{hn} - \frac{\mu\gamma_{hn}(\gamma_{hn}-\alpha)}{(1-\tilde{\lambda})\gamma_{hn} - \tilde{\lambda}\alpha + (1-\lambda)A}. \tag{78}$$

Note that $\dot{\gamma}_{hn}$ is decreasing in γ_{hn} and increasing in $\tilde{\lambda}$. Thus, $\dot{\gamma}_{hn} < 0$ for $\gamma_{hn} > \bar{\gamma}_{hn}$ and $\dot{\gamma}_{hn} > 0$ for $\gamma_{hn} < \bar{\gamma}_{hn}$, where $\bar{\gamma}_{hn}$ denotes the measure of buyers for which $\dot{\gamma}_{hn} = 0$. Moreover, note that $\bar{\gamma}_{hn}$ is decreasing on the transition path along which also high-valuation owners of lemons are sellers as $\dot{\tilde{\lambda}} \leq 0$. Thus, if γ_{hn}^o exceeds $\bar{\gamma}_{hn}$ at time 0, γ_{hn} decreases monotonically. Otherwise, $\gamma_{hn} \geq \gamma_{hn}^o$ throughout the transition path. In both cases, $\dot{\gamma}_{hn} > 0$ when high-valuation owners switch from selling to holding. This can by investigating the evolution of γ_{hn} when only low-valuation owners are sellers

$$\dot{\gamma}_{hn} = \nu - (\kappa+\nu)\gamma_{hn} - \frac{\mu\gamma_{hn}(\gamma_{hn}-\alpha)}{2\gamma_{hn}-\alpha}. \tag{79}$$

Note that this is strictly smaller than (78) for all γ_{hn}. Thus, $\bar{\gamma}_{hn} > \gamma_{hn}^a$ for all t. Given that $\gamma_{hn} \geq \gamma_{hn}^o > \gamma_{hn}^a$ or $\gamma_{hn} \geq \bar{\gamma}_{hn}$ for all t, it follows that $\dot{\gamma}_{hn} < 0$ at the point in time when high-valuation owners switch from selling to holding. Then, using $\bar{p}_{lg}(\bar{t}) - \bar{p}_{hg}(\bar{t}) = 0$ and $\bar{p}_{lg}^a - \bar{p}_{hg}^a = 0$ as in the proof of Proposition 4, one can establish that $\bar{p}_{lg}(\bar{t}) - \bar{p}_{hg}(\bar{t}) > 0$, constituting a contradiction. Moreover, note that Proposition 4 refers to a special case of this proposition with $\bar{t} = 0$. Then by continuity, there is an open set of parameter values for which neither a transition path along high-valuation owners of lemons switch from selling to holding nor a transition path along which only low-valuation owners are sellers exists. □

It is worth describing in words what has been established. Taken together, Propositions 3 and 5 show that, for some parameters, the following holds:

There exists a stationary equilibrium where all assets are traded and only low-valuation owners are sellers, limiting the severity of the lemons problem. However, trading of good assets can cease due to self-fulfilling expectations, in which case the market embarks on an equilibrium path along which only lemons are traded. From initial conditions determined by the limiting stationary equilibrium, there does not exist an equilibrium path back to the stationary equilibrium where all assets are traded and only low-valuation agents are sellers.

Although Propositions 3 and 5 already establish a strong path-dependence in equilibrium dynamics, it is of interest to investigate whether alternative equilibrium paths along which all assets are traded exist. More specifically, given that trade of all assets resuming induces additional owners to sell their asset, it is natural to investigate whether there exists a transition path along which all assets are traded and also high-valuation owners of lemons are sellers. It is worth emphasizing that a transition path to an altogether different stationary equilibrium is considered. Namely, to a stationary equilibrium where all assets are traded but also high-valuation owners of lemons are sellers. Let me address the question whether such a stationary equilibrium exists for parameter values supporting a self-fulfilling market freeze.

Consider whether it is optimal for a buyer to offer the high price when also high-valuation owners of lemons are sellers. When the pool of sellers with lemons expands, the expected surplus to a buyer from offering the high price decreases for two reasons. First, the fraction of sellers with good assets falls. Second, the rate at which a seller meets buyers decreases, leading to a fall in the value of owning a lemon. Thus, it may not be optimal for a buyer to offer the high price even though other buyers would do so when also high-valuation owners of lemons are sellers. Figure 7 shows that a stationary equilibrium where all assets are traded and also high-valuation owners of lemons are sellers does not exist for parameter values supporting a self-fulfilling market freeze. This can be seen from the fact a buyer obtains a higher expected surplus from offering the low price than the high price for $\lambda \in [\bar{\lambda}^a, \bar{\lambda}^a_{hb}]$. As in Figure 5, $\bar{\lambda}^a_{hb}$ indicates the fraction of good assets for which $\bar{p}^a_{lg} = \bar{p}^a_{hb}$. For $\lambda \leq \bar{p}^a_{hb}$, high-valuation owners of lemons prefer holding to selling in the stationary equilibrium where all assets are traded. Given that $\bar{\lambda}^a_{hb} < \bar{\lambda}^o$, the two stationary equilibria of Proposition 1 exist for $\lambda \in [\bar{\lambda}^a, \bar{\lambda}^a_{hb}]$.

For parameter values for which there does not exist a stationary equilibrium where all assets are traded and also high-valuation owners of lemons are sellers, no transition path to such an equilibrium exists either. Thus, Figure 7 also illustrates that, for some parameter values, a market freeze can be a trap from which

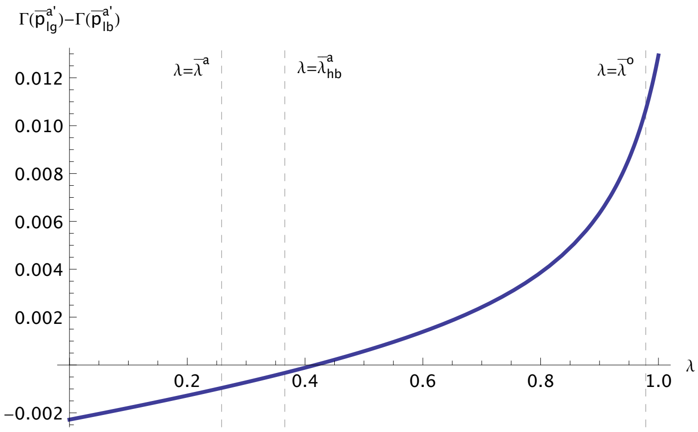

Figure 7: Buyer's expected surplus in the candidate stationary equilibrium where all assets are traded and also high-valuation owners of lemons are sellers (a'). Parameter values are as in Figure 5.

no transition path along which all assets are traded exists.[22] This is due to the fact that $\Gamma(\bar{p}^{a'}_{lb}) > \Gamma(\bar{p}^{a'}_{lg})$ for $\lambda \leq \bar{\lambda}^a_{hb}$.[23] Given that Proposition 5 applies for λ below and sufficiently close to $\bar{\lambda}^a_{hb}$, all equilibrium paths along which all assets are traded can be ruled out.

To summarize, a market freeze results in an accumulation of sellers of good assets. Consequently, if trade of all assets resumes, high-valuation owners of lemons may wish to sell their asset and enter the pool of buyers. If additional sellers of lemons enter the market, the fraction of good assets in the market and the rate at which a seller meets buyers fall. Thus, buyers may be unwilling to switch to offering high prices, accepted by sellers of both types.

To assess the robustness of the findings in this section, let me highlight the features of the environment which support a permanent market freeze. First, when $\nu > \kappa A$, there are more buyers than sellers and trade of the good assets ceasing leads to an increase in the rate at which a buyer meets sellers. Consequently, if trade of all assets resumed, high-valuation owners of lemons would have a stronger incentive to sell their asset. However, their entry into the pool of sellers increases the severity of the lemons problem, making recovery from a market freeze more difficult to attain. Second, the constant returns to scale

[22]What has been established is that there does not exist a recovery path from a market freeze when restricting attention to equilibria in symmetric pure strategies. However, the fact that V_{lb} decreases when only a fraction of buyers offer the high price p_{lg}, lowering the expected surplus from offering p_{lg}, suggests that there does not exist a recovery path in asymmetric or mixed strategies either.

[23]The superscript a' denotes a stationary equilibrium where all assets are traded and also high-valuation owners of lemons are sellers.

matching function allows market tightness to determine the rate at which buyers and sellers meet. Thus, if the measure of sellers increases relatively more than that of buyers, the rate at which a seller meets buyers falls. This, in turn, makes buyers less willing to acquire an asset of unknown quality as they anticipate difficulty in finding a buyer in the future. If, for instance, the increasing returns matching function in Duffie et al. (2005) was employed, a market freeze would increase the ease of finding a counterparty for both a seller and for a buyer. This would increase the value of a low-quality asset and support recovery from a market freeze as an equilibrium.

5 Conclusion

This paper shows that a decentralized asset market subject to adverse selection can support strongly asymmetric equilibrium dynamics. Namely, trading of good assets can cease due to self-fulfilling expectations. The resulting partial market freeze, on the other hand, can be a trap from which no equilibrium path along which all assets are traded exists. This path-dependence arises for the following reason. When only lemons are traded, the fraction of sellers offering good assets increases. Moreover, due to an accumulation of sellers, the rate at which a buyer meets sellers rises. As a result, if trade of all assets resumed, the value of being a buyer would be elevated. Thus, even owners of lemons attaching a high value to their asset's dividend flow, would be induced to sell their asset and enter the pool of sellers. Consequently, the average quality of assets in the market would fall, rendering buyers reluctant to switch to offering high prices, which would resume the trade of all assets.

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